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One-loop Effective Action for Spherical Scalar Field Collapse

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ABSTRACT

We calculate the complete one-loop effective action for a spherical scalar field collapse in the large radius approximation. This action gives the complete trace anomaly, which beside the matter loop contributions, receives a contribution from the graviton loops. Our result opens a possibility for a systematic study of the back-reaction effects for a real black hole.

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1. Introduction

In reference [1] a background-field formalism has been set up for calculating the complete one-loop effective action for a generic 2d dilaton gravity whose potential has a certain asymptotic behavior. This asymptotics was taken because it appears in the 2d dilaton gravity models which describe the spherical general relativity [2, 3], as well as in the CGHS model of 2d black holes [4]. Therefore the effective action derived in [1] can describe the back-reaction effects for a realistic 4d black hole. However, the matter in [1] does not couple to the dilaton, so that the action derived there corresponds to a spherical null-dust collapse, which is not the most realistic model of collapse, although it is useful, since the classical equations of motion can be integrated [5], and consequently one can develop an operator quantization by using the techniques developed for the CGHS case [6, 7, 8].

In this paper we apply the formalism of [1] to the case when the dilaton is coupled to the matter, in order to obtain a one-loop effective action for a spherical scalar field collapse. In ref. [9] a similar method has been applied to the case of general matter-dilaton coupling, but only the divergent part of the effective action has been calculated.

We start from the Einstein-Hilbert action in 4d with a minimally coupled scalar field f

$$S = \int d^4x \sqrt{-g_4} \left(\frac{1}{16\pi G} R_4 - \frac{1}{2} (\nabla_4 f)^2 \right) . \quad (1.1)$$

By using a spherically symmetric reduction ansatz [2]

$$ds^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu + e^{-2\Phi} d\Omega^2 , \quad (1.2)$$

one obtains

$$S = \int d^2x \sqrt{-\tilde{g}} e^{-2\Phi} \left(\tilde{R} + 2(\tilde{\nabla}\Phi)^2 + 2e^{2\Phi} - \frac{1}{2}(\nabla f)^2 \right) , \quad (1.3)$$

where we have set the Newton constant G to one. For the purpose of obtaining the semi-classical limit, it is useful to replace the scalar field with N scalar fields, so we will consider the action

$$S = \int d^2x \sqrt{-\tilde{g}} \left[e^{-2\Phi} \left(\tilde{R} + 2(\tilde{\nabla}\Phi)^2 + 2e^{2\Phi} \right) - \frac{1}{2} \sum_i e^{-2\Phi} (\tilde{\nabla} f_i)^2 \right] . \quad (1.4)$$

The calculation of the effective action simplifies if one performs a conformal transformation $\tilde{g}_{\mu\nu} = e^\Phi g_{\mu\nu}$, which gives

$$S = \int d^2x \sqrt{-g} \left[\phi R + 2\phi^{-\frac{1}{2}} - \frac{1}{2} \phi \sum_i (\nabla f_i)^2 \right] , \quad (1.5)$$

where $\phi = e^{-2\Phi}$. The action (1.5) will be our classical action.

2. Background field method

The one loop effective action for the classical action (1.5) can be found by using the background field method developed in [1]. In [1] a one-loop effective action for a spherical null-dust collapse has been found, in the limit of large radius $r = \sqrt{\phi} = e^{-\Phi} \gg 1$. Since the spherical null-dust action differs from (1.5) only in the ϕ -dependent matter coupling, the corresponding calculation for (1.5) is going to be almost identical, except for appropriate modifications due to the ϕ -dependent matter coupling in (1.5).

The one-loop effective action will be given by

$$\Gamma_1[\phi_0] = S(\phi_0) - \frac{1}{2i} \text{Tr} (\log S''(\phi_0)) \quad , \quad (2.1)$$

where $S''(\phi_0)$ is the second functional derivative of the classical action evaluated for the set of classical background fields $\phi_0 = \{g_{\mu\nu}, \phi, f_0\}$. The corresponding quantum fields are denoted as $\{h_{\mu\nu}, \hat{\phi}, f\}$. We choose the same gauge-fixing condition as in the null-dust case

$$\chi_\mu = D_\lambda h_\mu^\lambda - \frac{1}{2} D_\mu h - \frac{1}{\phi} D_\mu \hat{\phi} = 0 \quad , \quad (2.2)$$

which produces the same gauge-fixing term in the action

$$S_{GF} = -\frac{1}{2} \int d^2x \sqrt{-g} \phi \chi_\mu \chi^\mu \quad . \quad (2.3)$$

The effect of (2.3) is that the new action has a minimal structure, i.e. the second spacetime derivatives acting on the quantum fields appear only as \square . We also rescale the quantum fields as

$$h_{\mu\nu} \rightarrow \frac{h_{\mu\nu}}{\sqrt{\phi}} \quad , \quad \hat{\phi} \rightarrow \sqrt{\phi} \hat{\phi} \quad , \quad f \rightarrow \frac{f}{\sqrt{\phi}} \quad , \quad (2.4)$$

in order to remove the ϕ dependence from the kinetic terms for the quantum fields. The Jacobian of the transformation (2.4) is equal to 1. After that we use the t'Hooft-Veltman complexification of fields [10] and take the spacetime dimension to be $D = 2 + \epsilon$, in order to be able to use the dimensional regularization procedure. The quadratic part of the action is then given by

$$S_{tot}^{(2)} = \frac{1}{2} \int d^2x \sqrt{-g} (\bar{h}^{*\mu\nu} h^* \hat{\phi}^* f^*) \hat{K} (I \square + \hat{K}^{-1} \hat{M}) \begin{pmatrix} \bar{h}_{\rho\sigma} \\ h \\ \hat{\phi} \\ f \end{pmatrix} \quad , \quad (2.5)$$

where $I = \text{diag}(\Pi_{\rho\sigma}^{\mu\nu}, 1, 1, 1)$, $\bar{h}_{\mu\nu} = \Pi_{\mu\nu}^{\rho\sigma} h_{\rho\sigma} = h_{\mu\nu} - \frac{1}{D} h g_{\mu\nu}$ and

$$\hat{K} = \begin{pmatrix} \Pi_{\mu\nu}^{\rho\sigma} & 0 & 0 & 0 \\ 0 & -\frac{\epsilon}{2(2+\epsilon)} & -1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad (2.6)$$

$$\hat{K}^{-1} \hat{M} = \begin{pmatrix} \hat{V}_{\alpha\beta}^{\rho\sigma} & \hat{G}_{\alpha\beta} & \hat{H}_{\alpha\beta} & \hat{W}_{\alpha\beta} \\ \hat{M}^{\rho\sigma} & \hat{P} & \hat{Q} & \hat{X} \\ \hat{N}^{\rho\sigma} & \hat{L} & \hat{S} & \hat{E} \\ \hat{Y}^{\rho\sigma} & \hat{Z} & \hat{O} & \hat{F} \end{pmatrix}. \quad (2.7)$$

The matrix elements in (2.7) are given by

$$\begin{aligned} \hat{V}_{\alpha\beta}^{\rho\sigma} = & -\Pi_{\alpha\beta}^{\rho\sigma} R + \Pi_{\alpha\beta}^{\mu\nu} (R_{\mu}^{\rho} \delta_{\nu}^{\sigma} + R_{\mu}^{\sigma} \delta_{\nu}^{\rho} - R_{\mu\nu}^{\rho\sigma} - R_{\nu\mu}^{\rho\sigma}) \\ & - 3\Pi_{\alpha\beta}^{\rho\sigma} \square \Phi + 7\Pi_{\alpha\beta}^{\rho\sigma} (\nabla \Phi)^2 + 2D_{\lambda} \Phi (\Pi_{\alpha\beta}^{\sigma a} g^{\lambda\rho} + \Pi_{\alpha\beta}^{\rho a} g^{\lambda\sigma}) \\ & - \Pi_{\alpha\beta}^{\lambda\sigma} g^{a\sigma} - \Pi_{\alpha\beta}^{\lambda\rho} g^{a\sigma} \overrightarrow{D}_a + 4\Pi_{\alpha\beta}^{\mu\nu} (\delta_{\mu}^{\rho} D^{\sigma} D_{\nu} \Phi \\ & - 2\delta_{\mu}^{\rho} D^{\sigma} \Phi D_{\nu} \Phi + \delta_{\mu}^{\sigma} D^{\rho} D_{\nu} \Phi - 2\delta_{\mu}^{\sigma} D^{\rho} \Phi D_{\nu} \Phi) \\ & - \Pi_{\alpha\beta}^{\mu\nu} (\delta_{\mu}^{\rho} D_{\nu} f_0 D^{\sigma} f_0 + \delta_{\mu}^{\sigma} D_{\nu} f_0 D^{\rho} f_0) + \frac{1}{2} \Pi_{\alpha\beta}^{\rho\sigma} (\nabla f_0)^2 - 2\Pi_{\alpha\beta}^{\rho\sigma} \phi^{-\frac{3}{2}}, \end{aligned} \quad (2.8)$$

$$\begin{aligned} \hat{G}_{\alpha\beta} = & \Pi_{\alpha\beta}^{\mu\nu} \left(\frac{2-\epsilon}{2+\epsilon} R_{\mu\nu} + 2\frac{3\epsilon-2}{2+\epsilon} D_{\mu} \Phi D_{\nu} \Phi - 2\frac{\epsilon-2}{2+\epsilon} D_{\mu} D_{\nu} \Phi + 2D_{\mu} \Phi \overrightarrow{D}_{\nu} \right) \\ & + \frac{\epsilon-2}{2(2+\epsilon)} \Pi_{\alpha\beta}^{\mu\nu} D_{\mu} f_0 D_{\nu} f_0, \end{aligned} \quad (2.9)$$

$$\hat{H}_{\alpha\beta} = \Pi_{\alpha\beta}^{\mu\nu} (D_{\mu} f_0 D_{\nu} f_0 - 2R_{\mu\nu}), \quad (2.10)$$

$$\hat{W}_{\alpha\beta} = 2\Pi_{\alpha\beta}^{\mu\nu} (1 + e^{\Phi}) \partial_{\nu} f_0 \overrightarrow{\partial}_{\mu} + 2\Pi_{\alpha\beta}^{\mu\nu} D_{\mu} f_0 D_{\nu} \Phi, \quad (2.11)$$

$$\begin{aligned} \hat{M}^{\rho\sigma} = & \frac{2\epsilon}{1+\epsilon} R^{\rho\sigma} - \frac{2+\epsilon}{1+\epsilon} \left(2\frac{3\epsilon-2}{2+\epsilon} D^{\rho} \Phi D^{\sigma} \Phi - 2\frac{\epsilon-2}{\epsilon+2} D^{\rho} D^{\sigma} \Phi + (\overleftarrow{\partial}^{\rho} D^{\sigma} \Phi + \overleftarrow{\partial}^{\sigma} D^{\rho} \Phi) \right) \\ & - \frac{\epsilon}{1+\epsilon} D^{\rho} f_0 D^{\sigma} f_0, \end{aligned} \quad (2.12)$$

$$\begin{aligned} \hat{P} = & -\frac{\epsilon^2}{(1+\epsilon)(2+\epsilon)} R - \frac{2+\epsilon}{1+\epsilon} \left(-D^{\mu} \Phi \overrightarrow{\partial}_{\mu} - \frac{-\epsilon^2 + 5\epsilon + 6}{(2+\epsilon)^2} \square \Phi \right. \\ & \left. + \frac{-4\epsilon^2 + 3\epsilon + 6}{(2+\epsilon)^2} (\nabla \Phi)^2 + \frac{7\epsilon-2}{2(2+\epsilon)} \phi^{-\frac{3}{2}} \right) + \frac{\epsilon^2}{2(1+\epsilon)(2+\epsilon)} (\nabla f_0)^2, \end{aligned} \quad (2.13)$$

$$\hat{Q} = \frac{\epsilon}{2(1+\epsilon)} ((\nabla f_0)^2 - 2R) + \frac{2+\epsilon}{1+\epsilon} (2\overleftarrow{\partial}^{\mu} D_{\mu} \Phi + 2\square \Phi + 2(\nabla \Phi)^2 - \frac{1}{2} \phi^{-\frac{3}{2}}), \quad (2.14)$$

$$\hat{X} = (2 - e^\Phi \frac{\epsilon}{1+\epsilon}) \partial_\mu f_0 \vec{\partial}^\mu + 2D^\nu \Phi D_\nu f_0, \quad (2.15)$$

$$\begin{aligned} \hat{N}^{\rho\sigma} = & -\frac{1}{1+\epsilon} R^{\rho\sigma} - \frac{2+\epsilon}{2(1+\epsilon)} \left(2\frac{3\epsilon-2}{2+\epsilon} D^\rho \Phi D^\sigma \Phi - 2\frac{\epsilon-2}{\epsilon+2} D^\rho D^\sigma \Phi \right. \\ & \left. + (\overleftarrow{\partial}^\rho D^\sigma \Phi + \overleftarrow{\partial}^\sigma D^\rho \Phi) \right) + \frac{1}{2(1+\epsilon)} D^\rho f_0 D^\sigma f_0, \end{aligned} \quad (2.16)$$

$$\begin{aligned} \hat{L} = & \frac{\epsilon}{2(2+\epsilon)(1+\epsilon)} R - \frac{\epsilon}{2(1+\epsilon)} D^\mu \Phi \vec{\partial}_\mu + \frac{2-\epsilon}{2+\epsilon} \square \Phi \\ & - \frac{-3\epsilon^2 + 6\epsilon + 8}{2(1+\epsilon)(2+\epsilon)} (\nabla \Phi)^2 - \frac{\epsilon}{4(1+\epsilon)(2+\epsilon)} (\nabla f_0)^2 - \frac{9\epsilon}{4(1+\epsilon)} \phi^{-\frac{3}{2}}, \end{aligned} \quad (2.17)$$

$$\hat{S} = -\frac{\epsilon}{2(1+\epsilon)} R + \frac{2+\epsilon}{1+\epsilon} (\overleftarrow{\partial}^\mu D_\mu \Phi + \frac{1}{2} \phi^{-\frac{3}{2}}) + \frac{1}{1+\epsilon} \left(\square \Phi + (\nabla \Phi)^2 + \frac{\epsilon}{4} (\nabla f_0)^2 + \frac{3\epsilon}{4} \phi^{-\frac{3}{2}} \right), \quad (2.18)$$

$$\hat{E} = \frac{\epsilon}{2(1+\epsilon)} e^\Phi \partial^\mu f_0 \vec{\partial}_\mu, \quad (2.19)$$

$$\hat{Y}^{\rho\sigma} = \overleftarrow{\partial}^\rho (e^\Phi + 1) \partial^\sigma f_0 + D^\rho f_0 D^\sigma \Phi, \quad (2.20)$$

$$\hat{Z} = -\frac{\epsilon}{2(2+\epsilon)} \overleftarrow{\partial}^\rho (1 + e^\Phi) \partial_\rho f_0 - \frac{\epsilon}{2(2+\epsilon)} D^\rho \Phi D_\rho f_0, \quad (2.21)$$

$$\hat{O} = -\overleftarrow{\partial}^\rho \partial_\rho f_0 - D^\rho f_0 D_\rho \Phi, \quad (2.22)$$

$$\hat{F} = \square \Phi - (\nabla \Phi)^2. \quad (2.23)$$

The novel features in the spherical scalar case are that \hat{O} and \hat{F} matrix elements are non-zero, while the other matrix elements are modified by the terms coming from the dilaton-matter coupling.

After fixing of the quantum gauge symerties, we must add the ghost action to the action (2.5). The ghost action is the same as in the null-dust case [1]

$$S_{gh} = \int d^2x \sqrt{-g} \phi \bar{c}^\mu [-\square c_\mu - R_\mu^\nu c_\nu + \frac{1}{\phi} D_\mu (c^\rho \partial_\rho \phi)] \quad (2.24)$$

In (2.24) we have omitted the terms which do not contribute to the one-loop effective action. We then rescale the ghosts as

$$\phi \bar{c}^\mu \rightarrow \bar{c}^\mu, \quad (2.25)$$

so that

$$S_{gh} = \int d^2x \sqrt{-g} \bar{c}^\mu [-\square c_\mu - R_\mu^\nu c_\nu + \frac{1}{\phi} D_\mu (c^\rho \partial_\rho \phi)] \quad (2.26)$$

3. Expansion around a flat metric

The calculation of the one-loop effective action can be simplified by expanding the background metric around a flat metric $\eta_{\mu\nu}$ as

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu} + O(\gamma^2) \quad . \quad (3.1)$$

After inserting (3.1) into (2.6) and (2.7), we get

$$\sqrt{-g}(I\Box + \hat{K}^{-1}\hat{M}) = \text{diag}(P_{\alpha\beta}^{\rho\sigma}, 1, 1, 1)\partial^2 + K^{-1}M, \quad (3.2)$$

where $\partial^2 = \eta^{ab}\partial_a\partial_b$, $P_{\alpha\beta}^{\rho\sigma}$ is given in (3.8) and

$$K^{-1}M = \begin{pmatrix} \tilde{V}_{\alpha\beta}^{\rho\sigma} & G_{\alpha\beta} & H_{\alpha\beta} & W_{\alpha\beta} \\ M^{\rho\sigma} & P & Q & X \\ N^{\rho\sigma} & L & S & E \\ Y^{\rho\sigma} & Z & O & F \end{pmatrix} \quad . \quad (3.3)$$

The matrix elements in (3.3) which are relevant for our calculation are given by

$$\begin{aligned} \tilde{V}_{\alpha\beta}^{\rho\sigma} &= \overleftarrow{\partial}_a A_{\alpha\beta}^{ab\rho\sigma} \overrightarrow{\partial}_b \\ &- \left(\eta^{ab} P_{\epsilon\tau}^{\rho\sigma} S_{b\alpha\beta}^{\epsilon\tau} - 2\sqrt{-g}\partial_\lambda \Phi (g^{\lambda\rho} \Pi_{\alpha\beta}^{\sigma a} + \Pi_{\alpha\beta}^{\rho a} g^{\lambda\sigma} - \Pi_{\alpha\beta}^{\lambda\sigma} g^{a\rho} - \Pi_{\alpha\beta}^{\lambda\rho} g^{a\sigma}) \right) \overrightarrow{\partial}_a \\ &+ \overleftarrow{\partial}_a \eta^{ab} P_{\alpha\beta}^{\epsilon\tau} S_{b\tau\epsilon}^{\rho\sigma} - \sqrt{-g}\Pi_{\alpha\beta}^{\rho\sigma} R + \sqrt{-g}\Pi_{\alpha\beta}^{\mu\nu} (R_\mu^\rho \delta_\nu^\sigma + R_\mu^\sigma \delta_\nu^\rho - R_{\mu\nu}^{\rho\sigma} - R_{\nu\mu}^{\rho\sigma}) \\ &- 3\sqrt{-g}\Pi_{\alpha\beta}^{\rho\sigma}\Box\Phi + 7\sqrt{-g}\Pi_{\alpha\beta}^{\rho\sigma}(\nabla\Phi)^2 + \\ &+ 4\sqrt{-g}\Pi_{\alpha\beta}^{\mu\nu} \left(\delta_\mu^\rho D^\sigma D_\nu \Phi - 2\delta_\mu^\rho D^\sigma \Phi D_\nu \Phi + \delta_\mu^\sigma D^\rho D_\nu \Phi - 2\delta_\mu^\sigma D^\rho \Phi D_\nu \Phi \right) \\ &- 2\sqrt{-g}\partial_\lambda \Phi (\Pi_{\alpha\beta}^{a\tau} g^{\lambda\epsilon} + \Pi_{\alpha\beta}^{\epsilon a} g^{\lambda\tau}) \\ &- \Pi_{\alpha\beta}^{\lambda\epsilon} g^{a\tau} - \Pi_{\alpha\beta}^{\lambda\tau} g^{a\epsilon}) S_{a\epsilon\tau}^{\rho\sigma} + \eta^{ab} P_{\delta\eta}^{\epsilon\tau} S_{a\alpha\beta}^{\delta\eta} S_{b\epsilon\tau}^{\rho\sigma} \\ &- \sqrt{-g}\Pi_{\alpha\beta}^{\mu\nu} (\delta_\mu^\rho D_\nu f_0 D^\sigma f_0 + \delta_\mu^\sigma D_\nu f_0 D^\rho f_0) + \frac{1}{2}\sqrt{-g}\Pi_{\alpha\beta}^{\rho\sigma} ((\nabla f_0)^2 - 4\phi^{-\frac{3}{2}}), \end{aligned} \quad (3.4)$$

$$\begin{aligned} P &= \overleftarrow{\partial}_a \bar{\gamma}^{ab} \overrightarrow{\partial}_b - \frac{\epsilon^2}{(1+\epsilon)(2+\epsilon)} \sqrt{-g} R - \frac{2+\epsilon}{1+\epsilon} \sqrt{-g} \left(-D^\mu \Phi \overrightarrow{\partial}_\mu \right. \\ &\quad \left. - \frac{-\epsilon^2 + 5\epsilon + 6}{(2+\epsilon)^2} \Box\Phi + \frac{-4\epsilon^2 + 3\epsilon + 6}{(2+\epsilon)^2} (\nabla\Phi)^2 + \frac{7\epsilon - 2}{2(2+\epsilon)} \phi^{-\frac{3}{2}} \right) \\ &\quad + \frac{\epsilon^2}{2(1+\epsilon)(2+\epsilon)} \sqrt{-g} (\nabla f_0)^2, \end{aligned} \quad (3.5)$$

$$\begin{aligned} S &= \overleftarrow{\partial}_a \bar{\gamma}^{ab} \overrightarrow{\partial}_b - \frac{\epsilon}{2(1+\epsilon)} \sqrt{-g} (R - \frac{1}{2}(\nabla f_0)^2) + \frac{4+5\epsilon}{4(1+\epsilon)} \phi^{-\frac{3}{2}} \\ &\quad + \sqrt{-g} \frac{1}{1+\epsilon} \left((2+\epsilon) \overleftarrow{\partial}^\mu D_\mu \Phi + \Box\Phi + (\nabla\Phi)^2 \right), \end{aligned} \quad (3.6)$$

$$F = \overleftarrow{\partial}_a \bar{\gamma}^{ab} \overrightarrow{\partial}_b + \sqrt{-g}(\square\Phi - (\nabla\Phi)^2) \quad , \quad (3.7)$$

where

$$\begin{aligned} P_{\rho\sigma}^{\mu\nu} &= \frac{1}{2}(\delta_\rho^\mu \delta_\sigma^\nu + \delta_\sigma^\mu \delta_\rho^\nu) - \frac{1}{D}\eta^{\mu\nu}\eta_{\rho\sigma}, \\ S_{a\mu\nu}^{\rho\sigma} &= 2\Gamma_{a(\mu}^{(\rho} \delta_{\nu)}^{\sigma)}, \\ \bar{\gamma}^{\mu\nu} &= \gamma^{\mu\nu} - \frac{1}{2}\gamma\eta^{\mu\nu}, \\ A_{\alpha\beta}^{\rho\sigma ab} &= P_{\alpha\beta}^{\rho\sigma}\bar{\gamma}^{ab} - \frac{1}{D}\eta^{ab}(\gamma^{\rho\sigma}\eta_{\alpha\beta} - \gamma_{\alpha\beta}\eta^{\rho\sigma}) \quad , \end{aligned} \quad (3.8)$$

and $\gamma = \gamma^{\mu\nu}\eta_{\mu\nu}$. In the case of the ghost action (2.26), the expansion (3.1) yields [1]

$$\begin{aligned} S_{gh} &= \int d^2x \bar{c}^\mu (\delta_\mu^\nu \partial^2 + T_\mu^\nu) c_\nu \\ &= \int d^2x \bar{c}^\mu \left[\delta_\mu^\nu \partial^2 + \delta_\mu^\nu \overleftarrow{\partial}_a \bar{\gamma}^{ab} \overrightarrow{\partial}_b - \Gamma_{a\mu}^\nu \eta^{a\sigma} \overrightarrow{\partial}_\sigma + \eta^{ab} \Gamma_{a\mu}^\rho \Gamma_{b\rho}^\nu \right. \\ &\quad + \overleftarrow{\partial}_\sigma \eta^{a\sigma} \Gamma_{a\mu}^\nu + 2\partial_\rho \Phi (\eta^{\rho\nu} - \bar{\gamma}^{\rho\nu}) \overrightarrow{\partial}_\mu - 2\eta^{\rho\alpha} \Gamma_{\alpha\mu}^\nu \partial_\rho \Phi + \sqrt{-g} R_\mu^\nu \\ &\quad \left. - 2\delta^{\rho\nu} \Gamma_{\rho\mu}^\alpha \partial_\alpha \Phi - (\eta^{\rho\nu} - \bar{\gamma}^{\rho\nu})(4\partial_\mu \Phi \partial_\rho \Phi - 2\partial_\mu \partial_\rho \Phi) \right] c_\nu \quad . \end{aligned} \quad (3.9)$$

The one-loop correction to the effective action is then given by

$$\Gamma_1 = \frac{i}{2} \text{Tr} \log \left(1 + K^{-1} M \frac{1}{\partial^2} \right) - i \text{Tr} \log \left(1 + T \frac{1}{\partial^2} \right), \quad (3.10)$$

where $K^{-1}M$ and T are defined by (3.3) and (3.9). After expanding the logarithm in (3.10), we obtain

$$\begin{aligned} \Gamma_1 &= \frac{i}{2} \text{tr} \left[(\tilde{V}_{\alpha\beta}^{\rho\sigma} P_{\rho\sigma}^{\alpha\beta} + P + S + F) \frac{1}{\partial^2} - \frac{1}{2} (\tilde{V}_{\alpha\beta}^{\rho\sigma} \frac{1}{\partial^2} P_{\mu\nu}^{\alpha\beta} \tilde{V}_{\gamma\delta}^{\mu\nu} \frac{1}{\partial^2} P_{\rho\sigma}^{\gamma\delta} \right. \\ &\quad + 2G_{\alpha\beta} \frac{1}{\partial^2} P_{\mu\nu}^{\alpha\beta} M^{\mu\nu} \frac{1}{\partial^2} + 2H_{\alpha\beta} \frac{1}{\partial^2} P_{\mu\nu}^{\alpha\beta} N^{\mu\nu} \frac{1}{\partial^2} + \\ &\quad + 2Y_{\alpha\beta} \frac{1}{\partial^2} P_{\mu\nu}^{\alpha\beta} W^{\mu\nu} \frac{1}{\partial^2} + P \frac{1}{\partial^2} P \frac{1}{\partial^2} + S \frac{1}{\partial^2} S \frac{1}{\partial^2} \\ &\quad + F \frac{1}{\partial^2} F \frac{1}{\partial^2} + 2Q \frac{1}{\partial^2} L \frac{1}{\partial^2} + 2X \frac{1}{\partial^2} Z \frac{1}{\partial^2} + 2E \frac{1}{\partial^2} O \frac{1}{\partial^2} \left. \right] \\ &\quad - i \text{tr} \left[T_\nu^\mu \delta_\mu^\nu \frac{1}{\partial^2} - \frac{1}{2} T_\nu^\mu \delta_\rho^\nu \frac{1}{\partial^2} T_\sigma^\rho \delta_\mu^\sigma \frac{1}{\partial^2} \right] \quad , \end{aligned} \quad (3.11)$$

where tr denotes the spacetime trace, and symmetric ordering has been taken in the vertices, i.e. $v(x)p \rightarrow \frac{1}{2}(v(x)p + pv(x))$ where $p = i\partial_x$.

4. One-loop diagrams and the effective action

As in the null-dust case [1], we will calculate (3.11) by evaluating it for $g_{\mu\nu} = \eta_{\mu\nu}$ and for $\Phi = \text{constant}$ and then add these contributions to the contribution which vanishes in these special cases.

The contribution due to $\bar{h}_{\mu\nu}$ in the loops can be written as

$$\begin{aligned} tr \left(\tilde{V} \frac{1}{\partial^2} - \frac{1}{2} \tilde{V} \frac{1}{\partial^2} \tilde{V} \frac{1}{\partial^2} \right) &= tr \left[(A + B + C) \frac{1}{\partial^2} \right] \\ &\quad - \frac{1}{2} tr \left(A \frac{1}{\partial^2} A \frac{1}{\partial^2} + B \frac{1}{\partial^2} B \frac{1}{\partial^2} + C \frac{1}{\partial^2} C \frac{1}{\partial^2} \right) \\ &\quad - tr \left(A \frac{1}{\partial^2} B \frac{1}{\partial^2} + A \frac{1}{\partial^2} C \frac{1}{\partial^2} + B \frac{1}{\partial^2} C \frac{1}{\partial^2} \right) \quad . \quad (4.1) \end{aligned}$$

In (4.1), we denote the vertices with two, one and zero spacetime derivatives as A, B, and C, respectively. We will also refer to terms $tr(X \frac{1}{\partial^2})$ and $tr(X \frac{1}{\partial^2} Y \frac{1}{\partial^2})$ as diagrams X and XY respectively, where X and Y are any of the vertices.

It is easy to see that $A = B = 0$ after the infrared regularization (see the appendix of [1]). The C diagram is given by

$$\begin{aligned} C &= -\frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \int d^2x \left[\left(\frac{-D^2 + D + 2}{2} - \frac{4}{D} \right) \sqrt{-g} R + \eta^{ab} P_{\delta\eta}^{\epsilon\tau} P_{\rho\sigma}^{\alpha\beta} S_{a\alpha\beta}^{\delta\eta} S_{b\epsilon\tau}^{\rho\sigma} \right] \\ &\quad + 4 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \frac{D+2}{2} \int d^2x \sqrt{-g} R \Phi \\ &\quad - \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} (D^2 + D - 2) \Gamma(-\epsilon/2) \left(\left(\frac{7}{2} - \frac{8}{D} \right) \int d^2x \sqrt{-g} [(\nabla\Phi)^2 - \frac{N+2}{2} \square\Phi] \right. \\ &\quad \left. + \left(\frac{1}{4} - \frac{1}{D} \right) N \int d^2x \sqrt{-g} (\nabla f_0)^2 - \int d^2x \sqrt{g} \phi^{-\frac{3}{2}} \right) . \quad (4.2) \end{aligned}$$

The term with f_0 in (4.2) is new, and appears due to the dilaton-matter coupling. The factor N comes from the N scalar fields. As explained in [1], the AA diagram is given by

$$- \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \frac{D^2 + D - 2}{2} \Gamma(1 - \frac{\epsilon}{2}) B(2 + \frac{\epsilon}{2}, 2 + \frac{\epsilon}{2}) \int d^2x \sqrt{-g} \left(R \frac{1}{\square} R + \frac{4}{\epsilon(1 + \frac{\epsilon}{2})} R \right) , \quad (4.3)$$

while the AB diagram vanishes. The AC diagram is given by

$$AC = 2 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \int d^2x \sqrt{-g} \left(R \frac{1}{\square} R + \frac{N}{2} R \frac{1}{\square} (\nabla f_0)^2 + R \frac{1}{\square} (\nabla\Phi)^2 - R \Phi \right) . \quad (4.4)$$

For the BB diagram we get

$$BB = -8 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \left(\int d^2x \sqrt{-g} R \frac{1}{\square} R + 4 \int d^2x \sqrt{-g} R \right)$$

$$\begin{aligned}
& -2 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \int d^2x [\eta^{ab} P_{\delta\eta}^{\epsilon\tau} P_{\rho\sigma}^{\alpha\beta} S_{\alpha\beta}^{\delta\eta} S_{b\epsilon\tau}^{\rho\sigma} - 2(1-\epsilon)(D+2)\sqrt{-g}R\Phi] \\
& + 4 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) B(1+\frac{\epsilon}{2}, 1+\frac{\epsilon}{2}) (D^2 + D - 2) \int d^2x \sqrt{-g} (\nabla\Phi)^2. \quad (4.5)
\end{aligned}$$

The non-covariant terms in (4.2) and (4.5) vanish. The BC and CC diagrams are infrared divergent, but after an appropriate regularization [1], they also vanish.

The contribution to the effective action from the P and PP diagrams are the same as in the null-dust case. This is a consequence of the fact that $P = O(\epsilon^2)$. On the other hand, the S and the F diagram have a non-zero contribution to the effective action. If we denote as X a diagram in the set $\{P, S, F\}$, then

$$\begin{aligned}
\sum_X \text{tr} \left(X \frac{1}{\partial^2} - \frac{1}{2} X \frac{1}{\partial^2} X \frac{1}{\partial^2} \right) = & -\frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \frac{4\epsilon^2 - 3\epsilon - 6}{(1+\epsilon)(2+\epsilon)} \int d^2x \sqrt{-g} (\nabla\Phi)^2 \\
& - \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \left(\frac{1}{1+\epsilon} - \frac{1}{2} \left(\frac{2+\epsilon}{1+\epsilon} \right)^2 \right) \int d^2x \sqrt{-g} (\nabla\Phi)^2 \\
& - 4 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \int d^2x \sqrt{-g} \square\Phi \\
& + \frac{N+2}{2} \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(1-\frac{\epsilon}{2}) B(2+\frac{\epsilon}{2}, 2+\frac{\epsilon}{2}) \left(\int d^2x \sqrt{-g} R \frac{1}{\square} R \right. \\
& \left. + \frac{4}{\epsilon(1+\frac{\epsilon}{2})} \int d^2x \sqrt{-g} R \right) + \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \int d^2x \sqrt{-g} R \Phi \\
& - 2 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \int d^2x \sqrt{-g} R \frac{1}{\square} (\nabla\Phi)^2 + N \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \int d^2x \sqrt{-g} (\nabla\Phi)^2 \\
& + \frac{N}{2} \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \int d^2x \sqrt{-g} \left(-2R \frac{1}{\square} (\nabla\Phi)^2 + 2R\Phi + (\nabla f_0)^2 \right) \\
& + \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \frac{9\epsilon - 8}{4(1+\epsilon)} \int d^2x \sqrt{g} \phi^{-\frac{3}{2}}. \quad (4.6)
\end{aligned}$$

The diagrams XZ and EO are combinations of $(\nabla f_0)^2$ and $e^\Phi (\nabla f_0)^2$, and they can be neglected in the large-radius limit. The diagram WY is divergent, and it is given by

$$\frac{D^2 + D - 2}{2D} N \frac{i\pi^{\frac{D}{2}}}{4\pi^2} \Gamma(-\frac{\epsilon}{2}) B(1+\frac{\epsilon}{2}, 1+\frac{\epsilon}{2}) \int d^2x \sqrt{-g} (\nabla f_0)^2.$$

The diagrams GM and QL are the same as in the null-dust case, and they are given by

$$GM = -4 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \frac{2+\epsilon}{1+\epsilon} \left(\frac{1}{2} \Gamma(-\frac{\epsilon}{2}) B(1+\frac{\epsilon}{2}, 1+\frac{\epsilon}{2}) \frac{D^2 + D - 2}{2D} \right)$$

$$\begin{aligned}
& -\Gamma(1-\epsilon/2)(1-1/D)) \cdot \int d^2x \sqrt{-g} (\nabla\Phi)^2 \quad , \\
QL &= 2 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \int d^2x \sqrt{-g} (\nabla\Phi)^2 \quad .
\end{aligned} \tag{4.7}$$

The contribution to the effective action due to the ghost loops is given by

$$\begin{aligned}
tr \left(T \frac{1}{\partial^2} - \frac{1}{2} T \frac{1}{\partial^2} T \frac{1}{\partial^2} \right) &= tr \left[(\bar{A} + \bar{B} + \bar{C}) \frac{1}{\partial^2} \right] \\
&\quad - \frac{1}{2} tr \left(\bar{A} \frac{1}{\partial^2} \bar{A} \frac{1}{\partial^2} + \bar{B} \frac{1}{\partial^2} \bar{B} \frac{1}{\partial^2} + \bar{C} \frac{1}{\partial^2} \bar{C} \frac{1}{\partial^2} \right) \\
&\quad - tr \left(\bar{A} \frac{1}{\partial^2} \bar{B} \frac{1}{\partial^2} + \bar{A} \frac{1}{\partial^2} \bar{C} \frac{1}{\partial^2} + \bar{B} \frac{1}{\partial^2} \bar{C} \frac{1}{\partial^2} \right) \quad .
\end{aligned} \tag{4.8}$$

The diagrams which appear in (4.8) are the same as in the null-dust case, so that

$$\bar{A} = \bar{B} = \bar{A}\bar{B} = \bar{B}\bar{C} = 0, \tag{4.9}$$

$$\begin{aligned}
\bar{C} &= -\frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \int d^2x \left(\sqrt{-g} R + \eta^{ab} \Gamma_{ab}^\rho \Gamma_{b\rho}^\nu + \Phi \square \gamma \right) \\
&\quad + \frac{4i\pi^{\frac{D}{2}}}{(2\pi)^2} \Gamma(-\frac{\epsilon}{2}) \int d^2x \sqrt{-g} [(\nabla\Phi)^2 - \frac{1}{2} \square \Phi],
\end{aligned} \tag{4.10}$$

$$\begin{aligned}
\bar{A}\bar{A} &= D \int d^2x d^2y \bar{\gamma}^{ab}(x) \bar{\gamma}^{cd}(y) \partial_a^x \partial_d^y G(y-x) \partial_b^x \partial_c^y G(x-y) \\
&= -\frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} D \Gamma(1-\frac{\epsilon}{2}) B(2+\frac{\epsilon}{2}, 2+\frac{\epsilon}{2}) \int d^2x \sqrt{-g} \left(R \frac{1}{\square} R + \frac{4}{\epsilon(1+\frac{\epsilon}{2})} R \right),
\end{aligned} \tag{4.11}$$

$$\bar{A}\bar{B} = \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \int d^2x \sqrt{-g} R \Phi, \tag{4.12}$$

$$\bar{A}\bar{C} = -\frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \int d^2x \sqrt{-g} \left(R \frac{1}{\square} R - 4R \frac{1}{\square} (\nabla\Phi)^2 + 2R\Phi \right), \tag{4.13}$$

$$\begin{aligned}
\bar{B}\bar{B} &= -2 \frac{i\pi^{\frac{D}{2}}}{(2\pi)^2} \left(\int d^2x \sqrt{-g} (R \frac{1}{\square} R + 4R - 2R\Phi) \right. \\
&\quad \left. + \Gamma(-\frac{\epsilon}{2}) \int d^2x (\Gamma_{c\mu}^\nu \Gamma_{d\nu}^\mu \eta^{cd} + \Phi \square \gamma + \sqrt{-g} (\nabla\Phi)^2) \right).
\end{aligned} \tag{4.14}$$

From (3.11), (4.2-14) we get the bare effective action

$$\begin{aligned}
\bar{\Gamma}_1 &= S - \frac{N-24}{96\pi} \int d^2x \sqrt{-g} R \frac{1}{\square} R - \frac{N-24}{24\pi\epsilon} \int d^2x \sqrt{-g} R - \frac{1}{2\pi} \int d^2x \sqrt{-g} R \frac{1}{\square} (\nabla\Phi)^2 \\
&\quad - \frac{\pi^{\frac{\epsilon}{2}}}{8\pi} [-8\Gamma(-\epsilon/2) + 23] \int d^2x \sqrt{-g} (\nabla\Phi)^2 + \frac{5}{4\pi} \int d^2x \sqrt{-g} R \Phi \\
&\quad - \frac{N\pi^{\frac{\epsilon}{2}}}{8\pi} \Gamma(-\frac{\epsilon}{2}) \int d^2x \sqrt{-g} ((\nabla\Phi)^2 - \square\Phi) + \frac{\pi^{\frac{\epsilon}{2}}}{4\pi} \Gamma(-\frac{\epsilon}{2}) \int d^2x \sqrt{-g} (\square\Phi - \phi^{-\frac{3}{2}}) \\
&\quad - \frac{N\pi^{\frac{\epsilon}{2}}}{8\pi} \int d^2x \sqrt{-g} \left(R\Phi - R \frac{1}{\square} (\nabla\Phi)^2 - R \frac{1}{\square} (\nabla f_0)^2 \right),
\end{aligned} \tag{4.15}$$

in the large-radius approximation, where S is the classical action given by (1.5). The divergent part of the action (4.15) agrees with the result of [9], up to boundary terms. After making a modified minimal subtraction of the poles in (4.15) we get the renormalized one-loop effective action

$$\begin{aligned} \Gamma_1 = S - \frac{N-24}{96\pi} \int d^2x \sqrt{-g} R \frac{1}{\square} R - \frac{1}{\pi} \int d^2x \sqrt{-g} \left(\frac{1}{2} R \frac{1}{\square} (\nabla \Phi)^2 - \frac{5}{4} R \Phi + \frac{23}{8} (\nabla \Phi)^2 \right) \\ - \frac{N}{8\pi} \left(\int d^2x \sqrt{-g} (R \Phi - R \frac{1}{\square} (\nabla \Phi)^2 - R \frac{1}{\square} (\nabla f)^2) \right) + O(e^{2\Phi}) \quad , \end{aligned} \quad (4.16)$$

where we have denoted f_0 as f .

The expression (4.16) is our final result. Since the spherically reduced models are good approximation for describing the quantum effects of massive black holes ($M \gg M_{Pl}$) and for $r \gg r_{Pl}$, then by taking the large- N limit of (4.16) and neglecting the $O(r^{-2})$ terms, we obtain

$$\Gamma'_1 = S - \frac{N}{96\pi} \int d^2x \sqrt{-g} \left(R \frac{1}{\square} R - 12 R \frac{1}{\square} (\nabla \Phi)^2 + 12 R \Phi - 12 R \frac{1}{\square} (\nabla f)^2 \right) \quad . \quad (4.17)$$

Note that the matter loops contribution is given by the first three terms in (4.17), while the last term comes from the graviton loops which are induced by a non-zero coupling between the matter and the dilaton. The result (4.17) can be rewritten in the black hole metric (1.2) as

$$\begin{aligned} \Gamma'_1 = S - \frac{N}{96\pi} \int d^2x \sqrt{-g} \left(R \frac{1}{\square} R - 12 R \frac{1}{\square} (\nabla \Phi)^2 + 13 R \Phi + \square \Phi \frac{1}{\square} R - 12 R \frac{1}{\square} (\nabla f)^2 \right. \\ \left. - (\nabla \Phi)^2 + 12 \Phi \square \Phi - 12 \square \Phi \frac{1}{\square} (\nabla f)^2 - 12 \square \Phi \frac{1}{\square} (\nabla \Phi)^2 \right) . \end{aligned} \quad (4.18)$$

After performing partial integrations one obtains a simpler form

$$\begin{aligned} \Gamma'_1 = S - \frac{N}{96\pi} \int d^2x \sqrt{-g} \left(R \frac{1}{\square} R - 12 R \frac{1}{\square} (\nabla \Phi)^2 + 14 R \Phi - 12 R \frac{1}{\square} (\nabla f)^2 \right. \\ \left. - 13 (\nabla \Phi)^2 - 12 \Phi (\nabla f)^2 - 12 \Phi (\nabla \Phi)^2 \right) \quad , \end{aligned} \quad (4.19)$$

but given that in the case of the collapse geometry there is a non-trivial boundary, these two forms will differ by boundary terms.

5. Conclusions

Note that recently two papers have appeared [11, 12], where the conformal factor dependent part of the effective action for a spherical scalar has been computed. This

part of the action gives the trace anomaly, and their results can be written in our normalization as

$$W = -\frac{N}{96\pi} \int d^2x \sqrt{-g} \left(R \frac{1}{\square} R - 12R \frac{1}{\square} (\nabla\Phi)^2 + c_3 R\Phi \right) \quad , \quad (5.1)$$

where $c_3 = -4$ in [11], while $c_3 = 12$ in [12]. Our result for W (the first line of (4.19)) differs from (5.1) by the presence of $R(1/\square)(\nabla f)^2$ term. This can be explained by the fact that in [11] only the matter loops have been taken into account, while in [12] the graviton loops have not been taken into account.

Our analysis implies that the trace anomaly part of the one-loop effective action is given by

$$W = -\frac{N}{96\pi} \int d^2x \sqrt{-g} \left(c_1 R \frac{1}{\square} R + c_2 R \frac{1}{\square} (\nabla\Phi)^2 + c_3 R\Phi + c_4 R \frac{1}{\square} (\nabla f)^2 \right) \quad , \quad (5.2)$$

where from (4.19) it follows that $c_1 = 1$, $c_2 = -12$, $c_3 = 14$ and $c_4 = -12$. Note that the value of c_3 is ambiguous, because the term $R\Phi$ is equivalent to $\square\Phi \frac{1}{\square} R$ up to boundary terms. Therefore the trace anomaly T can be found from

$$2 \frac{dW[kg_{\mu\nu}]}{dk} \Big|_{k=1} = \int d^2x \sqrt{-g} T \quad ,$$

so that

$$T = \frac{N}{24\pi} \left(R - 6(\nabla\Phi)^2 + \frac{1}{2}c_3 \square\Phi - 6(\nabla f)^2 \right) \quad . \quad (5.3)$$

Note that if one sets $N = 1$ in (4.16) instead of taking the large- N limit, the corresponding trace-anomaly part will be again of the form (5.2), but now the c_i coefficients will be different from the values obtained from (5.3) for $N = 1$. In particular, c_1 will be negative ($c_1 = 1 - 24 = -23$), due to the ghost contribution $c_g = -26$, which is the well-known 2d conformal anomaly. This is a generic situation for all relevant 2d dilaton models [1]. The resolution of this paradox has been suggested in [1], where it was pointed out that a resummation of the diagrams is a possible way of obtaining the same result as in the large- N limit. This was based on the fact that in the CGHS case one can calculate the exact one-loop effective action by using the reduced phase space quantization [7]. One then obtains the BPP action [13], which differs from the RST action [14] by a $(\nabla\Phi)^2$ term, and $c_1 = 1$, $c_2 = 0$ and $c_3 = 4$ in both cases. The exact action is the same as the large- N limit action, which coincides with the matter loops contribution. From the covariant perturbation theory point of view, this can be explained by the fact that the ghosts serve to cancel the loops containing the pure gauge degrees of freedom, which in the 2d dilaton gravity case are the graviton and the dilaton. Hence it should be possible to perform a resummation

of the gauge-field and the ghost loop diagrams such that one obtains the same values for the $c_{1,2,3}$ coefficients as those from the large-N action (in the null-dust case $c_1 = 1$, $c_2 = 0$ and $c_3 = 2$ [1]). We expect that essentially the same mechanism should work for the spherical scalar case. However, due to a non-zero dilaton-matter coupling the large-N limit is not the same as the matter loops contribution, and therefore c_4 is non-zero. Also note that for conformally invariant theories the coefficient of the pole in the divergent part of the one-loop action is the same as the finite part. In our case this property does not hold any more, since the classical theory is not conformally invariant, so that a non-zero value for c_4 is not prevented by a symmetry argument.

The value of the coefficient c_3 is apparently regularization-scheme dependent, although there is a further ambiguity in c_3 due to appearance of the term $\square\Phi\square^{-1}R$, which is the same as the $R\Phi$ term, up to boundary terms. Another potential source of ambiguity is the fact that we quantize the theory via (g, ϕ) variables, rather than via the original (\tilde{g}, Φ) variables. In principle the two quantum theories may differ, so that one can think that a non-zero value for c_4 may be related to this fact. One can show that within the one-loop background field formalism, an invertible field redefinition changes the effective action by the logarithm of the corresponding Jacobian. Calculating this Jacobian is difficult in general, because one must know the path-integral measure, which is not known in the general case. However, the calculations in the background-field formalism are done with trivial measures, so that a local field transformation induces the following Jacobian in our case

$$|J| = \exp\left(\alpha\delta(0)\int d^2x\sqrt{-g}\Phi\right) \quad , \quad (5.4)$$

where α is a constant. This expression is equal to one within the dimensional regularization, since $\delta(0) = 0$. More generally, one can expect that

$$|J| = \exp\left(\int d^2x\hat{O}\Phi\right) \quad , \quad (5.5)$$

where \hat{O} is an operator made from the metric and spacetime derivatives. The form of \hat{O} is constrained by the dimensionality of the effective action and the diffeomorphism invariance, so that

$$\hat{O} = \sqrt{-g}(\beta R + \gamma\square^{-1}R\square + \dots) \quad , \quad (5.6)$$

where β and γ are constants and \dots stand for higher-derivative terms. However, irrespective of the exact form of \hat{O} , (5.5) can only affect the value of c_3 . Therefore we expect that the value of c_4 stays the same. The best way to check this is to perform the corresponding calculation with (\tilde{g}, Φ) variables.

Given the complete one-loop effective action we have derived one can start investigating the solutions of the corresponding equations of motion in order to find the

back-reaction effect. An easier task would be to study the static vacuum solutions along the lines developed in [15], where the effective action had only the Polyakov-Liouville term.

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